# 3D Visualization of the Separated Fluid Flows 

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#### Abstract

For the detailed investigation of the 3D unsteady incompressible viscous separated fluid flows around a sphere (for $200 \leq R e \leq 700$ ) and a circular cylinder (for $200 \leq R e \leq 400$ ) the direct numerical simulation and 3 D visualization are used. For 3D visualization of the fluid flows around a sphere the definition of vortex core as a connected region containing two negative eigenvalues of the $S^{2}+\Omega^{2}$ tensor is used (where $S_{i, j}$ and $\Omega_{i, j}$ are the rate of strain and the rate of rotation tensors). The formation mechanism of vortices in the sphere wake for $R e=500$ is described in detail. For 3D visualization of the fluid flows around a circular cylinder the 3D isosurfaces of the streamwise component of vorticity $\omega_{x}$ are used.


Keywords: 3D Visualization, Sphere, Cylinder, Wake, Vortex structures

## 1. Introduction

The understanding of the dynamics and kinematics of the 3D unsteady separated fluid flows around the bluff bodies is very important both from theoretical and from practical point of view. With the development of high-performance computers and especially cost effective massive parallel computers with a distributed memory the numerical simulation becomes one of the more effective approaches for such investigations. The 3D velocity and pressure fields calculated by this numerical simulation can not explain us the dynamics and kinematics of the 3D unsteady separated fluid flows without the appropriate 3 D visualization of them.

Detailed investigations of the structure of the sphere wake in the homogeous fluid have been initiated in (Magarvey and Bishop, 1961) and prolongated in (Sakamoto and Haniu, 1990, 1995), (Gushchin and Matyushin, 1997), (Johnson and Patel, 1999), (Gushchin et al., 1998, 2001, 2002) and other papers. In spite of these papers, the detailed formation mechanism of vortices in the sphere wake is still unclear.

In the present work the direct numerical simulation is used for the detailed investigation of transitional (from 2D to 3D unsteady) and post transitional regimes of 3D separated incompressible viscous fluid flows around a sphere and a 3D circular cylinder. The governing Navier-Stokes equations are written as follows:

$$
\begin{align*}
& \frac{\partial \overrightarrow{\mathbf{v}}}{\partial t}+(\overrightarrow{\mathbf{v}} \bullet \nabla) \overrightarrow{\mathbf{v}}=-\nabla p+\frac{2}{\operatorname{Re} \Delta \overrightarrow{\mathbf{v}}}  \tag{1}\\
& \nabla \bullet \overrightarrow{\mathbf{v}}=0  \tag{2}\\
& \text { where } \overrightarrow{\mathbf{v}} \text { is velocity vector (non-dimensionalized by the uniform velocity } U \text { ), } p-\text { pressure }
\end{align*}
$$

(non-dimensionalized by the $\rho U^{2}$, where $\rho$ is fluid density ( $\rho=1$ )), $R e=U d / v$ is Reynolds number, $d$ is diameter of a sphere, $v$ is the coefficient of kinematic viscosity, distances are non-dimensionalized by the radius of the sphere $d / 2$, time is non-dimensionalized by the $d /(2 U)$. The boundary conditions imposed on $\overrightarrow{\mathbf{v}}$ are as follows: $\overrightarrow{\mathbf{v}}=0$ on the sphere; $\overrightarrow{\mathbf{v}}=(0,0,1)$ on the outer boundary.

For this direct numerical simulation the Splitting on physical factors Method for Incompressible Fluid flows (SMIF) with hybrid explicit finite difference scheme (second order of accuracy in space, minimum scheme viscosity and dispersion, capable for work in wide range of Reynolds numbers and monotonous) based on Modified Central Difference Scheme (MCDS) and Modified Upwind Difference Scheme (MUDS) with special switch condition depending on velocity sign and sign of the first and the second differences of transferred functions was developed and successfully applied (Gushchin and Konshin, 1992), (Gushchin and Matyushin, 1997). Some applications of SMIF for solving of different problems are described in (Belotserkovskii, 1997). The Poisson equation for the pressure was solved by the Preconditioned Conjugate Gradients Method. Each time step was chosen automatically from a scheme stability condition. The parallelization of the algorithm was made and successfully applied on massive parallel computers with a distributed memory such as PARAM 10000 (based on Ultra Sparc II processors ( 400 MHz )) and MBC 1000/17IAP (based on the Intel Xeon processors (1.7 GHz)).

In order to understand the dynamics and kinematics of the 3D unsteady separated fluid flows we must properly visualize the 3 D vortex structures of the wake. We need more than the 2D visualization of streamlines, pressure contours (isolines), vorticity contours in the some planes. In many cases the 3 D isosurfaces of pressure or of vorticity can not also give us the real vortex structures of the wake. A number of methods have been proposed for properly identifying vortical regions. Jeong and Hussain(1995) have reviewed these methods and have proposed a new method They identified the vortical regions by using the definition of vortex core as a connected region containing two negative eigenvalues of the $\mathbf{S}^{2}+\Omega^{2}$ tensor (where the rate of strain $\mathrm{S}_{\mathrm{i}, \mathrm{j}}$ and rate of rotation $\Omega_{\mathrm{i}, \mathrm{j}}$ tensors are $\left.\mathrm{S}_{\mathrm{i}, \mathrm{j}}=\left(\mathrm{v}_{\mathrm{i}, \mathrm{j}}+\mathrm{v}_{\mathrm{j}, \mathrm{i}}\right) / 2, \Omega_{\mathrm{i}, \mathrm{j}}=\left(\mathrm{v}_{\mathrm{i}, \mathrm{j}} \mathrm{v}_{\mathrm{j}, \mathrm{i}}\right) / 2\right)$. Jeong and Hussain provide number of examples to illustrate the advantages of this method over others, indicating a more robust and precise elucidation of the vortex regions. We are successfully applied this method for our 3D flow visualization (see Fig. 1 (a), where one quadrant of the vortex structure was cut away in order to see the sphere surface (blue color)).


Fig. 1. $R e=200$ : (a) Vortex Structure (Zero Isosurface of the Second Eigenvalues of the $\mathbf{S}^{2}+\Omega^{2}$ Tensor);
(b) Streamlines in the Symmetry Plane of the Wake.

## 2. 3D Visualization of the Flow around a Sphere

For the investigation of the 3D unsteady separated homogeneous viscous fluid flows around the sphere the spherical coordinate system (O-type grid) is used: $x=R \sin \theta \cos \varphi, y=R \sin \theta \sin \varphi$, $z=R \cos \theta$, where $z, x, y$ are streamwise, lift and lateral directions, accordingly (see Fig. 1 (a)). For appropriate approximation of the boundary layer the following transforming function in radius direction was used:

$$
\begin{equation*}
R_{i}=R\left(r_{i}\right)=1+\frac{i}{N_{0}} \sqrt{\frac{2}{\operatorname{Re}}}+\left(\frac{i}{N} r_{\max }\right)^{m} \dot{i}=1: N \quad m=3, r_{\max }=3, N=120, N_{0}=10 \tag{3}
\end{equation*}
$$

where $N_{0}$ is the number of grid points in the boundary layer in radial direction. The following number of grid points $(r, \theta, \varphi)$ is used: $(120 \times 60 \times 120)$. Methodical calculations for checking the grid dependency was carry out in (Matyushin, 2003) where was shown that for $R e \leq 500$ the reasonable grid independence is achieved on a $(120 \times 60 \times 120)$ grid with $N_{0}=10$.

At the present paper the classification of the 3D separated homogeneous viscous fluid flow regimes around the sphere at $0 \leq R e \leq 700$ was refined. For $0 \leq R e \leq 20.5$ the streamlines follow the sphere surface without separation. For $20.5 \leq R e \leq 270$ the separated fluid flows around a sphere are steady. For $20.5 \leq R e \leq 200$ the axisymmetrical separated fluid flows around a sphere were observed (see Fig. 1). The vortex structure in Fig. 1 (a) is divided into the recirculating zone of the wake (which is clearly seen in Fig. 1 (b) also) and a cylindrical separated shear layer (vortex sheet) at the periphery of the recirculating zone (which is invisible in Fig. 1 (b)). For $200<R e \leq 270$ the main axisymmetrical bubble (toroid) in the recirculating zone of the wake and cylindrical shear layer are deformed through a normal bifurcation in more topologically stable form (a double-thread wake (Fig. 2)). The 3D particle path (streamline) in Fig. 2 (b) reveals that there is a fluid which is flowing out from the centre of the upper part of the vortex in the recirculating zone of the wake and feeding into the centre of the lower part of this vortex through the core of the deformed toroid. Moreover the lower spiral (unstable focus) which is fed by fluid from the upper spiral (stable focus) releases fluid into the wake after sending it up and around the toroid. (In stable focus the liquid moves along a spiral towards its centre; in unstable focus the liquid moves along a spiral outwards its centre (Tobak and Peake, 1982).) In the Fig. 2 (b) you can see also skin friction patterns on the sphere surface which reveal primary separation line with two singular points (node and saddle) and rear stagnation point. The steady flows for $200<R e \leq 270$ are characterised by the existence of non-zero lift/side and torque moment coefficients (see (Magarvey and Bishop, 1961), (Johnson and Patel, 1999), (Gushchin et al., 1998, 2001, 2002)).


Fig. 2. $R e=211$ : (a) (top) Streamlines in the Symmetry Plane of the Wake.
(a) (bottom) Vortex Structure. (Zero Isosurface of the Second Eigenvalues of the $\mathbf{S}^{2}+\Omega^{2}$ Tensor)
(b) 3D Streamline and Skin Friction Patterns on the Sphere Surface: oblique and Y-Z Views

For $R e>270$ the wake becomes unsteady through a Hopf bifurcation and the separated fluid flows around a sphere are unsteady and periodical. For $270<R e<400$ periodical separation of the hairpin-shaped vortices is observed only from one (top) edge of cylindrical shear layer surrounding the recirculating zone (see Fig. 3), and the time-averaged lift/side and torque moment coefficients are not equal to zero. Besides for $360 \leq R e<400$ the regular rotation of the wake is observed (Strouhal numbers corresponding to this rotation are $S t_{\text {rot }}=0.0044,0.0058$ for $R e=375,380$ accordingly). $\left(S t_{\text {rot }}=f_{\text {rot }} d l U\right.$, where frot is the rotation frequency.) The Strouhal number for $R e=300$ is $S t=0.145$ ( $S t=f d l U$, where $f$ is the shedding frequency). For the same $R e=300$ the experimental results (Sakamoto and Haniu, 1990) give a Strouhal number range of 0.150-0.165 and the numerical result (Johnson and Patel, 1999) gives the value of 0.137 for Strouhal number. The vortex structure visualization in (Johnson and Patel, 1999) is very close to Fig. 3. Note that in Fig. 3 the vortex
structures show not only the hairpin vortices separated from the top of the sphere and facing upwards, but also additional vortices (facing downwards) induced by the interaction of the near wake flow and the outer flow.

For $R e \geq 400$ the periodical separation of the hairpin-shaped vortices is observed from opposite edges of the cylindrical shear layer alternatively and the time-averaged lift/side coefficients of such flows are equal to zero (see Figs. 4-7). For $R e>600$ the irregular rotation of the cylindrical shear layer, is observed.

The Strouhal numbers (see Table 1) obtained in this work are in a good agreement with experiment (Sakamoto and Haniu, 1990) ( $0.15<S t<0.2$ ) and other experimental and numerical results.


Fig. 3. $R e=300$ : Vortical Structures: (a) $t=775$, (b) $t=782$.
(Zero Isosurfaces of the Second Eigenvalues of the $\mathbf{S}^{2}+\Omega^{2}$ Tensor)
Table 1. Strouhal Numbers St, Time-averaged Lateral $C_{l}$ and Drag $C_{d}$ Coefficients versus $R e$.

| $R e$ | 280 | 290 | 300 | 350 | 360 | 375 | 380 | 390 | 400 | 500 | 600 | 700 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $S t$ | 0.133 | 0.140 | 0.145 | 0.141 | 0.160 | 0.191 | 0.193 | 0.154 | 0.135 | 0.112 | 0.142 | 0.150 |
| $C_{l}$ | 0.078 | 0.082 | 0.084 | 0.086 | 0.084 | 0.082 | 0.083 | 0.081 | 0.0 | 0.0 | 0.0 | 0.0 |
| $C_{d}$ | 0.683 | 0.675 | 0.669 | 0.636 | 0.630 | 0.622 | 0.619 | 0.615 | 0.607 | 0.582 | 0.561 | 0.536 |



Fig. 4. $R e=500$ : The total Lateral Coefficient Versus Time.


Fig. 5. $R e=500$ : Vortex Structures during a Half of the Period: $443<t<450$. (Zero Isosurfaces of the Second Eigenvalues of the $\mathbf{S}^{2}+\Omega^{2}$ Tensor)

The detail formation mechanism of vortices in the sphere wake for $R e \geq 400$ is shown in Figs. 5-7 where the separation of the hairpin-shaped vortex from bottom edge of the cylindrical shear layer (vortex sheet) during a half of the period ( $443<t<450$ ) is observed at $R e=500$. (Dimensionless time step $d t$ is equal to 0.0002.) In Fig. 6 the more detail views of the vortex structures from Fig. 5 are presented (one half of the vortex structure is removed for clarity). This formation mechanism can be divided in four stages. In Fig. 6 (a) (at the first stage) you can see the recirculating zone (in the vicinity of the sphere surface), the cylindrical shear layer (at the periphery of the recirculating zone) and the deformed vortex loop (facing upwards). The lower part of this deformed vortex loop belongs to
the lower part of the recirculating zone. Unlike flow regimes for $R e<400$ for this flow regime there is no deformed vortex ring in the recirculating zone (see Fig. 6). Therefore the upper and lower foci in Fig. 7 belong to the different vortex structures. The lower and upper foci in Figs. 6 (a) and 7 (a) belong to the lower and upper parts of the recirculating zone correspondingly.

At the second stage the bottom part of the deformed vortex loop (facing upwards) is extracted from the lower part of the recirculating zone (which is shifted closer to the sphere) and the bottom and side parts of the vortex sheet grow and undergo a stretching process (see Fig. 6 (b)). At the third stage the bottom edge of the vortex sheet is rolling up cylindrically and detached from the vortex sheet; the lower part of the recirculating zone is connected with the bottom part of the vortex sheet; the top and side parts of the deformed vortex loop are separated as hairpin-shaped vortex loop (facing upwards) (Fig. 6 (c)). At the fourth stage the bottom and side edges of the vortex sheet are transformed into the front and side parts of the nascent deformed vortex loop (facing downwards) (see Fig. 6 (d)).

During the next half of the period the front and side parts of the nascent deformed vortex loop (facing downwards) (see Fig. 6 (d)) are separated as hairpin-shaped vortex loop (facing downwards); the top and side edges of the vortex sheet are transformed into the front and side parts of the next nascent deformed vortex loop (facing upwards) and so on.


Fig. 6. $R e=500$ : Vortex Structures during a Half of the Period: $443<t<450$.(Zero Isosurfaces of the Second Eigenvalues of the $\mathbf{S}^{2}+\Omega^{2}$ Tensor).


Fig. 7. $R e=500$ : Streamlines in Symmetry Plane of the Wake: $443<t<450$.

## 3. The Visualization of the Vortex Structures in the Wake of a 3D Circular Cylinder

The transitional regimes of the separated homogeneous fluid flows around a 3D circular cylinder have been investigated in (König and Eckelmann, 1995), (Williamson, 1996), (Braza, 1998) and other papers. It was determined that flow in the wake of the 3D circular cylinder became 3 D for $R e>191$. In this paper transitional regimes of the separated homogeneous fluid flows around a 3D circular cylinder were investigated for $200 \leq R e \leq 400$. For this investigation the cylindrical coordinate system (O-type grid) is used: $x=R \cos \theta, y=R \sin \theta, z=z$, where $x, y, z$ are streamwise, lift and spanwize directions, accordingly. From the methodical calculations for 2D circular cylinder (see (Gushchin et al., 2002)) the following conclusions were made: for moderate Reynolds numbers $(100 \leq R e \leq 400)$ the grid in radial and circumferential directions should be at least $180 \times 180$ or $240 \times$ 240 , the number of the points in the boundary layer can be taken not less than 5 , the location of the outer boundary $23.78 d$ with usual ( $\overrightarrow{\mathbf{v}}=(1,0,0)$ ) or non-reflecting boundary conditions is sufficient to see the effects in the near wake region $(0.5 d<x<10 d$ ) ( $d$ is the diameter of the cylinder). The set of the 3 D calculations for the optimal flow parameters (grid size $(r, \theta, z)-240 \times 240 \times 72,10$ points in the boundary layer, location of the outer boundary - $23.78 d$, the length of the cylinder - $L=7.5 d$ ) was executed for $R e=220,240,260,280,300,320,340,360,400$. In addition the 3D calculations for the different length of the cylinder ( $3.5 d<L<15 d$ ) and different $R e$ numbers ( $200 \leq R e \leq 400$ ) were performed. Some examples of these combinations ( $L, R e$ ) are listed bellow: (3.5d, 250), (4.4d, 400), (7d, 230), (11.5d, 230).


Fig. 8. Isolines of the Streamwise Component of Vorticity $\omega_{x}$ in the Plane $y=0$ Mode A, $\operatorname{Re}=220$, L=7.5d.


Fig. 10. The Isosurfaces of the Streamwise Component of Vorticity $\omega_{x}$ Mode A, $\operatorname{Re}=240$, $L=7.5 \mathrm{~d}$.


Fig. 9. Isolines of the Streamwise Component of Vorticity $\omega_{x}$ in the Plane $y=0$ Mode $B, R e=320, L=7.5 d$.


Fig. 11. The Isosurfaces of the Streamwise Component of Vorticity $\omega_{x}$
Mode B, Re=320, L=7.5d.

We observe the mode A for $191 \leq R e \leq 300$ and mode B for $300 \leq R e \leq 400$. The isolines of the streamwise component of vorticity $\omega_{x}$ in the plane $y=0$ for modes A and B are shown in the Figs. 8 and 9 correspondingly $(\overrightarrow{\boldsymbol{\omega}}=\operatorname{rot} \overrightarrow{\mathbf{v}})$. In these figures the negative values of $\omega_{x}$ are green, the positive values are red. From these figures the periodicity along $z$ axis is easily seen. For mode A and B the periods along $z$ axis are equal to $3.5 d \leq \lambda \leq 4 d$ and $0.8 d \leq \lambda \leq 1.0 d$ correspondingly. It is worth noting
that vortex structures for mode A, obtained for different $R e$ numbers coincide qualitatively and distinguish by the intensity of vortex only. (Same for mode B).

The isosurfaces of the streamwise component of vorticity $\omega_{x}$ are shown in Figs. 10 and 11. Similar visualization was used in works (Williamson, 1996), (Braza, 1998). In this work for the first time the values of the maximum phase difference along the span were estimated. These values are approximately equal to $0.1-0.2 T$ (for mode A) and 0.015-0.030 $T$ (for mode B), where $T$ is the period of the flow. For $R e=300$ obtained both modes A and B are existing simultaneously.

In Figs. 12 and 13 the negative and positive values of the spanwize component of vorticity $\omega_{z}$ are visualized on the rear part of the surface of the cylinder (the $\theta$-coordinate starts from front stagnation line). The negative values of $\omega_{z}$ are blue, the positive values of $\omega_{z}$ are red. The separation and adhesion of the flow from the surface of the cylinder occurs on the lines with zero $\omega_{z}$.

In the Figs. 14 and 15 the streamlines in the $x-y$ planes for $z=0.73 d$ and $z=2.6 d$ are visualized at the same time. These pictures demonstrate the phase difference along the span of the cylinder. In Fig. 14 the joined vortex in the upper plane is already developed, but in the Fig. 15 the joined vortex in the upper plane is not developed yet.


Fig. 12. The Spanwize Component of Vorticity $\omega_{z}$ on the Surface of the Cylinder. Red are the Positive Values, Blue are the Negative Values. Mode A. $\operatorname{Re}=220, L=7.5 \mathrm{~d}$.


Fig. 14. The Streamlines in the Plane $\mathrm{Z}=0.73 \mathrm{~d}$, $\mathrm{Re}=260$, L=7.5d.


Fig. 13. The Spanwize Component of Vorticity $\omega_{z}$ on the Surface of the Cylinder. Red are the Positive Values, Blue are the Negative Values. Mode B. $\operatorname{Re}=320, \mathrm{~L}=7.5 \mathrm{~d}$.


Fig. 15. The Streamlines in the Plane $\mathrm{Z}=2.6 \mathrm{~d}$, $\mathrm{Re}=260$, L=7.5d.

## 4. Conclusion

The numerical method SMIF successfully applied in the present work showed a good comparison of our results with experimental and numerical works of the other researchers. The 3D visualization developed and used here gave us a possibility for more careful analysis of the transitional flow regimes around the sphere and the 3D circular cylinder and the further understanding and refining of classification of these flow regimes. For the first time in this paper the formation mechanism of vortices in the sphere wake for $R e \geq 400$ was described in detail and the values of the maximum phase difference along the span of 3D circular cylinder was estimated for modes A and B.

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## Author Profile



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